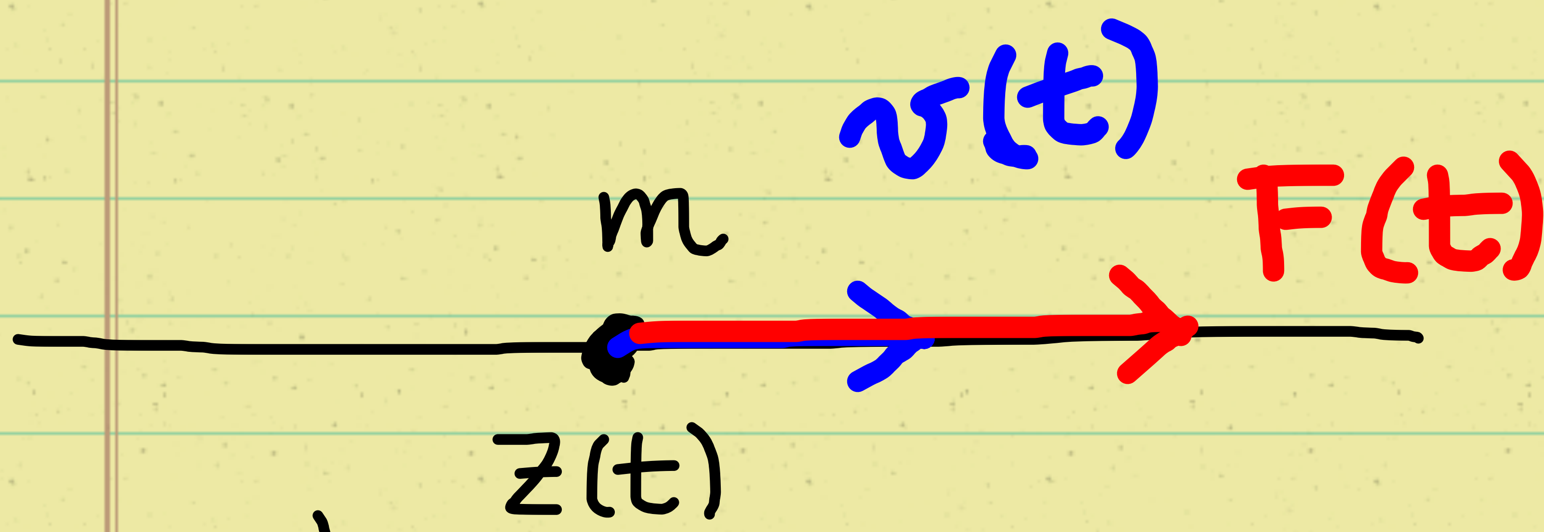
Tema 5: Trabajo y Energía.



$$F(t) = \phi(z(t))$$

(1) $z(t+\Delta t) - z(t) = v(t)\Delta t + \frac{1}{2} a_0 \Delta t^2$

(2) $v(t+\Delta t) - v(t) = a_0 \Delta t$

Supongamos que $F(t) = F_0$

$(1) \times F_0$

$$F_0 \left(z(t+\Delta t) - z(t)\right)$$

$$= F_0 v(t) \Delta t + \frac{1}{2} F_0 a_0 \Delta t^2$$

$$\boxed{\text{Si} \quad F_0 = m a_0}$$ Ley de Newton

$$= m\, a_0 \Delta t\, v(t) + \frac{1}{2m} m (a_0 \Delta t)^2$$

$$\boxed{\text{Recordemos} \quad v(t+\Delta t) - v(t) = a_0 \Delta t}$$

$$= m \left[v(t+\Delta t) - v(t) \right] v(t)$$

$$+ \frac{1}{2} m \left[v(t+\Delta t) - v(t) \right]^2$$

operando:

$$= \frac{1}{2} m v^2(t+\Delta t) - \frac{1}{2} m v(t)$$

En resumen.

$$\boxed{\begin{aligned} &F_0\left(z(t+\Delta t) - z(t)\right) \\ &= \frac{1}{2} m v^2(t+\Delta t) - \frac{1}{2} v^2(t) \end{aligned}}$$

Energía cinética $:= \frac{1}{2} m v^2(t)$

Trabajo $:= F_0\left(z(t+\Delta t) - z(t)\right)$

$W \cancel{= F_0\left(z(t+\Delta t) - z(t)\right)}$

$a+\Delta t \quad t_1 \quad t_2 \quad t_3 \quad t \quad t_4 \quad \cdots \quad b$

$a = t_0 \qquad b = t_n$

$W(t_{i+1}) - W(K_{i+1}) - K_i$

$$\boxed{\begin{aligned} &= F(t_i)\left(z(t_{i+1}) - z(t_i)\right) \\ &= \frac{1}{2} m v^2(t_{i+1}) - \frac{1}{2} m v^2(t_i) \end{aligned}}$$

$$W_{n,0} = W_{n,n-1} + W_{n-1,n-2} + \cdots + W_{1,0}$$

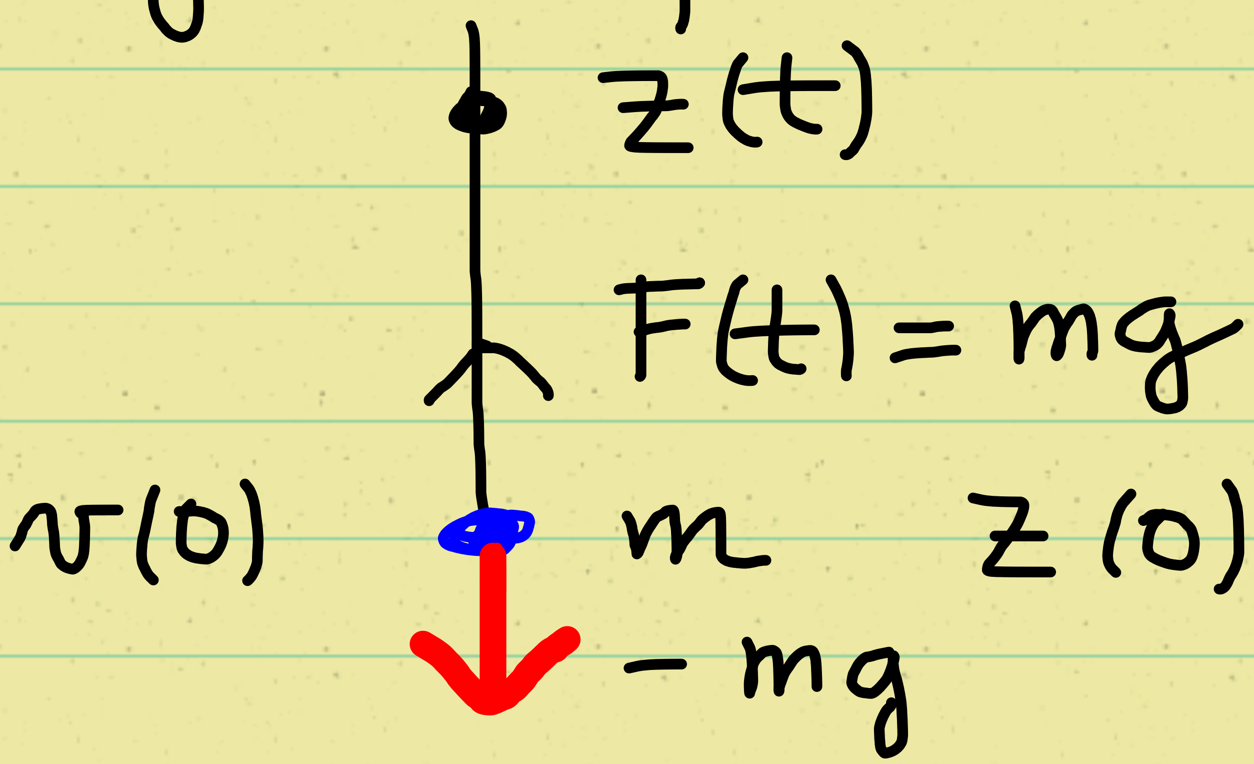
$$= \sum_{i=0}^{n} F(t_i)\left(z(t_{i+1}) - z(t_i)\right)$$

$$= \frac{1}{2} m v^2(b) - \frac{1}{2} m v^2(a)$$

Si $\Delta t \to 0$ se obtiene

$$\boxed{\int_a^b F\,dz = \frac{1}{2} m v^2(b) - \frac{1}{2} m v^2(a)}$$

Supongamos que



$$W_{t,0} = mg\left(z(t) - z(0)\right)$$
$$= \frac{1}{2} m v^2(t)$$

$$v(t) = \sqrt{2g(\underbrace{z(t) - z(0)}_{\text{altura}})}$$

que es la velocidad de caída libre

$$W_{b,a} = U(t_b) - U(t_a)$$

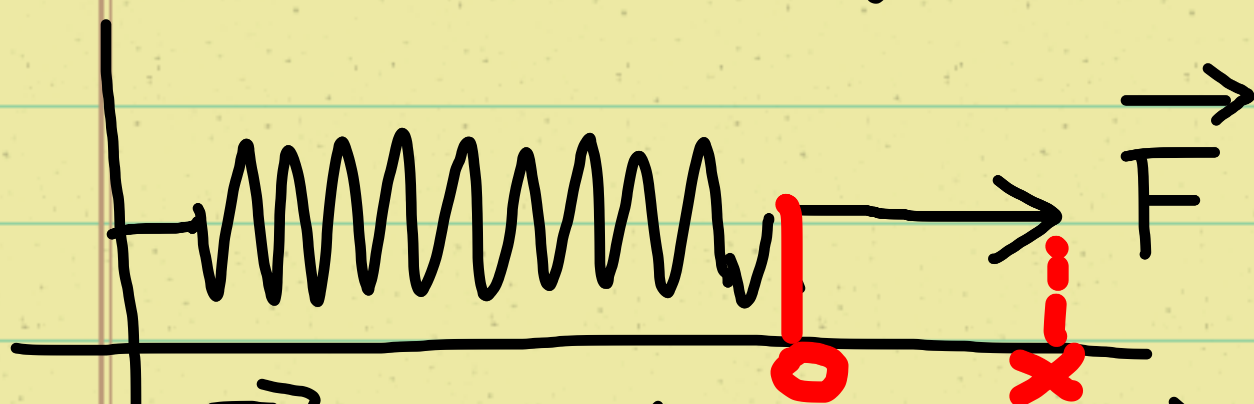
El trabajo viene producido por una variación de un potencial

$$W_{b,a} = mgz(t_b) - mgz(t_a)$$

$$U(t) = mgz(t)$$

potencial newtoniano o gravitatorio

Aplicaciones: Ley de Hooke



$$\vec{F}(x) = (kx, 0, 0)$$

$$W_{x_0,0} = \int_0^{x_0} kx\,dx = \frac{1}{2}kx_0^2$$